Active STAR-RIS Assisted Wireless Information and Power Transfer Systems

Jie Jiang, Bin Lyu, Pengcheng Chen, and Zhen Yang
School of Communications and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing 210003, China
Email: {jiangjie, blyu, 2020010104, yangz}@njupt.edu.cn

Abstract—In this paper, we propose an active simultaneously transmitting and reflecting reconfigurable intelligent surface (aSTAR-RIS) assisted wireless information and power transfer system. The aSTAR-RIS is employed to not only assist the downlink simultaneous wireless information and power transfer for the energy harvesting at the wireless powered devices (WPDs) and information receiving at the information receiving devices (IRDs), but also aid the uplink wireless information transmission from the WPDs to the hybrid access point. Our objective is to maximize the uplink sum-rate, considering the quality-of-service requirements of IRDs, by optimizing the power allocation, time scheduling, transmission and reflection beamforming at the aSTAR-RIS. To solve this non-convex problem, we propose an effective alternating optimization algorithm with semidefinite relaxation and fractional programming techniques. Numerical results demonstrate that the proposed aSTAR-RIS scheme can achieve up to 640.68% performance gains compared to the passive STAR-RIS scheme in our settings.

Index Terms—Wireless information and power transfer, reconfigurable intelligent surface (RIS), simultaneously transmitting and reflecting (STAR), active STAR-RIS, sum-rate maximization

I. INTRODUCTION

With the advance of ultra-dense Internet of Things (IoTs), wireless power transfer (WPT) has been put forth to satisfy power supply requirements for low-power devices [1]. Since radio frequency (RF) signals carry both energy and information, integrating WPT with wireless information transmission (WIT) can serve both the needs of communications and energy supplies [2]. Motivated by this, there are two typical research directions, i.e., simultaneous wireless information and power transfer (SWIPT) and wireless powered communication network (WPCN). In a SWIPT system, the downlink (DL) transmission of RF signals can simultaneously support information delivery to information receiving devices (IRDs) and energy transfer to wireless powered devices (WPDs) [3]. However, the uplink (UL) communication requirements for the SWIPT system are ignored. In a WPCN, there are only WPDs, which aim to harvest energy from the hybrid access point (HAP) first, and then use the harvested energy to transmit their information to the HAP [4]. It is obvious that the DL transmission of RF signals is not fully exploited, as only the functionality of WPT is utilized. To address these drawbacks suffered by SWIPT and WPCN, wireless information and power transfer (WIT) has been proposed [5], [6], which can satisfy both WPT and WIT in the DL and UL. Nevertheless, the practical implementation of WIPT systems is fundamentally restricted by short communication distance and instability due to severe path-loss and blockages. Moreover, the low efficiency of WET and WIT in WIPT systems results in a limited amount of harvested energy and achievable rate. As such, how to improve the performance of WIPT systems is a critical issue.

Recently, reconfigurable intelligent surface (RIS) has been regarded as an effective solution to modify the wireless radio environment and address blockage problems in wireless communication systems [7]. Based on a planner array equipped with a number of low-cost elements, RIS has been proven to be effective in terms of improving spectral efficiency (SE) [8], energy efficiency [9], ultra-reliable and safe connections [10], etc. Inspired by the superiority of the RIS, the integration of the RIS with WIPT systems has been investigated in [11]. The authors investigate a passive RIS empowered WIPT system, where a common HAP communicates with IRDs and WPDs with the assistance of two passive RISs. However, there are two drawbacks that need to be solved. First, the passive RIS can only passively reflect the incident signals without amplification, thereby suffering from the double-fading effect and resulting in a limited performance improvement. Second, the passive RIS can only reflect signals to one side of the surface, which limits the flexibility of network deployment.

To solve these drawbacks, a novel concept of active simultaneously transmitting and reflecting RIS (aSTAR-RIS) has been proposed [12], [13]. The STAR-RIS can simultaneously transmit and reflect incident signals to all devices positioned in both transmission and reflection spaces, resulting in full-space coverage [14]. Based on [15], three different operating protocols can be applied for the aSTAR-RIS, i.e., energy splitting (ES), time switching (TS), and mode switching. In addition, the aSTAR-RIS can further amplify the incident signals with the desired amplification gain [12]. Consequently, the amplification characteristics of the aSTAR-RIS can be exploited to improve information and energy transmission efficiency. However, to the best of our knowledge, no work has studied the application of aSTAR-RIS in WIPT systems.

In this paper, we study an aSTAR-RIS assisted WIT system, where the aSTAR-RIS is applied to not only assist the DL SWIPT for enabling the energy harvesting (EH) at the WPDs and information receiving at the IRDs, but also aid the UL WIT from the WPDs to the HAP. Specifically, for the DL SWIPT, the HAP transmits information-carrying...
signals via non-orthogonal multiple access (NOMA) to the IRDs and the WPDs harvest energy from the received signals in the meantime. The aSTAR-RIS exploits the ES protocol to improve the efficiency of DL SWIPT. For the UL WIT, each WPD transmits its own information signal to the HAP using the harvested energy via time division multiple access (TDMA). The aSTAR-RIS employs the TS protocol to avoid splitting signals from WPDs at the aSTAR-RIS because the signals only need to be transmitted to the HAP. In this setup, we then maximize the uplink sum-rate of the WPDs, taking into account the quality-of-service (QoS) requirements of IRDs, by jointly optimizing the transmit power of the HAP, the power allocation of the WPDs, time scheduling, transmission and reflection beamforming at the aSTAR-RIS. Since the formulated problem is non-convex and challenging to solve, we conceive an efficient alternating optimization (AO) algorithm with semidefinite relaxation (SDR) and fractional programming (FP) to solve it efficiently. Simulation results demonstrate that the proposed aSTAR-RIS scheme outperforms the passive STAR-RIS scheme in our considered situation.

II. SYSTEM MODEL

As depicted in Fig. 1, we consider an aSTAR-RIS assisted WIPT system, which comprises a single-antenna HAP, an aSTAR-RIS with $N$ elements, and two types of single-antenna devices, i.e., WPDs and IRDs. Due to the fact that the direct links between the HAP and WPDs/IRDs are blocked by the existing obstacles [15], the aSTAR-RIS is deployed to perform the DL SWIPT and support the UL WIT. Specifically, for the DL SWIPT, the IRDs receive the information-carrying signals from the HAP by exploiting the NOMA protocol. In the meantime, the WPDs harvest energy from these information-carrying signals. For the UL WIT, the WPDs utilize the harvested energy to send their own information to the HAP via TDMA. In this scenario, one pair of devices, consisting of a WPD and an IRD, is deployed in the reflection space ($r$-space), and the other pair is deployed in the transmission space ($t$-space). For simplicity, the devices are denoted by $U_{i,k}$, where $\forall i \in \{1, \ldots , N\}$ indicates that the device is an IRD or a WPD, and $\forall k \in \{r, t\}$ indicates that the device is deployed in the $r$-space or $t$-space, respectively. We assume that the pair of $U_{i,r}$ and $U_{p,r}$ is near the aSTAR-RIS and the other pair of $U_{i,t}$ and $U_{p,t}$ is far from the aSTAR-RIS [16]. The channels from HAP to aSTAR-RIS, aSTAR-RIS to $U_{i,k}$, $U_{p,k}$ to aSTAR-RIS, and aSTAR-RIS to the HAP are denoted by $h \in \mathbb{C}^{N \times 1}$, $h_{d,n}^{H} \in \mathbb{C}^{1 \times N}$, $g_{p,k} \in \mathbb{C}^{N \times 1}$, and $g^{H} \in \mathbb{C}^{1 \times N}$, respectively.

We assume that the channel state information (CSI) can be perfectly obtained to reveal the maximum performance gains provided by the aSTAR-RIS [13], [17], [18]. The channels are assumed to follow a quasi-static flat-fading model [19]. Moreover, only the first time of reflection/transmission at the aSTAR-RIS is considered due to the high path loss [11]. Furthermore, the durations of performing the DL SWIPT and UL WIT are denoted by $\tau_{r}$ and $\tau_{t}$, respectively. Besides, the UL WIT phase consists of two sub-slots with durations of $\tau_{r}$ and $\tau_{t}$, respectively. Then the whole time scheduling can be denoted by $\tau = [\tau_{d}, \tau_{r}, \tau_{t}]$.

A. Downlink SWIPT Phase

In this phase, the HAP transmits the superimposed signals to IRDs and WPDs with the assistance of the aSTAR-RIS. Since the STAR-RIS can simultaneously reflect and transmit signals, we utilize NOMA to transmit signals to the two IRDs in different spaces, which aims to greatly improve SE. In particular, the aSTAR-RIS working in the ES protocol simultaneously aids the WIT from the HAP to the IRDs and the WPT from the HAP to the WPDs in both spaces. Based on the ES protocol, the incident signals are split into transmitted and reflected parts [15]. The transmission/reflection coefficient matrices of the aSTAR-RIS in the DL phase are denoted by $\Theta_{d,k} = \text{diag}\left(\sqrt{\alpha_{d,1}} \beta_{d,1}^{k} e^{j\theta_{d,1}}, \ldots , \sqrt{\alpha_{d,N}} \beta_{d,N}^{k} e^{j\theta_{d,N}}\right)$, where $\alpha_{d,n} \in [0, \alpha_{\text{max}}]$, $\beta_{d,n}^{k} \in [0, 1]$, and $\theta_{d,n}^{k} \in [0, 2\pi]$ respectively represent the amplification gain, the amplitude coefficient, and the phase shift of the $n$-th element for $k$-space in the DL phase, $\forall k \in \{r, t\}$, $\forall n \in \mathcal{N} = \{1, \ldots , N\}$. It is worth noting that $\alpha_{\text{max}}$ can be greater than 1 since the active load can amplify the incident signals [20]. In addition, the amplitude coefficient of each element of the aSTAR-RIS is limited to $\beta_{d,n}^{k} + \beta_{d,n}^{r} = 1$, due to the law of energy conservation [21]. We assume that the phase shifts can be independently adjusted, which is widely adopted in the literature [15], [21], [22].

The superimposed signal at the HAP is expressed as $s_{d} = \sqrt{P_{d,r}}b_{d,r} + \sqrt{P_{d,t}}b_{d,t}$, where $s_{d,k} \in \mathbb{C}$ denotes the transmit signal with zero mean and unit variance for $U_{i,k}$. Let $P_{d}$ denote the transmitted power at the HAP and $p_{d,k}$ represent the transmitted power of the HAP for $U_{i,k}$, which satisfy $p_{d,r} + p_{d,t} = P_{d}$. Based on the NOMA protocol [23], the HAP allocates more energy to $U_{i,t}$ than $U_{i,r}$, i.e., $p_{d,t} \geq p_{d,r}$, to guarantee user fairness. Since $U_{i,r}$ is close to the HAP,
it is reasonable to consider that the channel gain between HAP and $U_{i,r}$ is better than that of HAP and $U_{i,t}$, i.e.,
$$\|h_{i,t}^H \Theta_d h_i\|^2 \geq \|h_{i,t}^H \Theta_d h_i\|^2 \|h_{i,t}^H \Theta_d h_i\|^2$$ [11]. Then, the received signal at $U_{i,k}$ is given by
$$y_{i,k} = h_{i,k}^H \Theta_d (h_{s_{d,k}} + n_r) + n_{i,k}, \quad (1)$$
where $n_{i,k} \sim \mathcal{CN} \left(0, \sigma^2_{i,k}\right)$ and $n_r \sim \mathcal{CN} \left(0, \sigma^2 I_N\right)$ represent the additive white Gaussian noise (AWGN) at $U_{i,k}$ and the aSTAR-RIS, respectively.

For this scenario, only $U_{i,r}$ applies the successive interference cancellation (SIC) to remove the interference, while $U_{i,t}$ decodes its received signal directly [23]. Specifically, $U_{i,r}$ first decodes the signal of $U_{i,t}$ and eliminates the interference by implementing the SIC, and then decodes its own signal. Accordingly, the signal to interference plus noise ratio (SINR) of $U_{i,t}$ to decode its signal, the SINR of $U_{i,r}$ to detect the signal of $U_{i,t}$, and the signal-noise-ratio (SNR) of $U_{i,r}$ to decode its own signal are respectively given by
$$\gamma_{i,t} = \frac{p_{d,t} \|h_{i,t}^H \Theta_d h_i\|^2}{p_{d,t} \|h_{i,t}^H \Theta_d h_i\|^2 + \sigma^2 \|h_{i,t}^H \Theta_d h_i\|^2 + \sigma^2_{i,t}}, \quad (2)$$
$$\gamma_{i,r} = \frac{p_{d,r} \|h_{i,r}^H \Theta_d h_i\|^2}{p_{d,r} \|h_{i,r}^H \Theta_d h_i\|^2 + \sigma^2 \|h_{i,r}^H \Theta_d h_i\|^2 + \sigma^2_{i,r}}, \quad (3)$$
$$\gamma_{i,t} = \frac{p_{d,t} \|h_{i,t}^H \Theta_d h_i\|^2}{\sigma^2 \|h_{i,t}^H \Theta_d h_i\|^2 + \sigma^2_{i,t}}. \quad (4)$$

Then, the achievable data rates of $U_{i,r}$ and $U_{i,t}$ can be respectively expressed as
$$R_{i,r} = \tau_d \log_2 (1 + \gamma_{i,r}), \quad (5)$$
$$R_{i,t} = \tau_d \log_2 (1 + \min \{\gamma_{i,t}, \gamma_{i,t}\}). \quad (6)$$

Since the WPDs only focus on harvesting energy in this phase, they can harvest energy from the superimposed signals transmitted by the HAP in the meantime. Thus, the received signal at $U_{p,k}$ is formulated as
$$y_{p,k} = h_{p,k}^H \Theta_d (h_{s_{d,k}} + n_r) + n_{p,k}, \quad (7)$$
where $n_{p,k} \sim \mathcal{CN} \left(0, \sigma^2_{p,k}\right)$ is the AWGN at $U_{p,k}$. Considering the two-piece linear EH model [24], the harvested energy by $U_{p,k}$ is modeled as
$$E_{h,k} = \tau_d \min \{\eta P_{i,n,k} + P_{\text{sat}}\}, \quad (8)$$
where $P_{i,n,k} = P_d \|h_{p,k}^H \Theta_d h_i\|^2 + \sigma^2 \|h_{p,k}^H \Theta_d h_i\|^2$ is the received power from the RF signals and the aSTAR-RIS noise, $\eta \in (0, 1]$ denotes the energy conversion efficiency of each WPD, and $P_{\text{sat}}$ is the saturation power (i.e., the maximum harvestable power).

Due to the limited power budget at the aSTAR-RIS, the maximum amplification power in the DL is limited by the following constraint
$$\sum_k P_d \|\Theta_d h_i\|^2 + \sigma^2 \|\Theta_d h_i\|^2 F_r \leq P_a, \quad (9)$$
where $P_a$ is the total amplification power.

**B. Uplink WIT Phase**

In the UL phase, the WPDs transmit their own information signals to the HAP by utilizing the harvested energy via TDMA. Considering the HAP is located on one side of the aSTAR-RIS, each WPD signal incident on the aSTAR-RIS does not need to be split into two parts. Therefore, the aSTAR-RIS adopts the TS protocol to exploit the time domain, in which all elements are switched to reflection mode during $\tau_r$ and transmission mode during $\tau_t$. The corresponding transmission/reflection coefficient of the aSTAR-RIS during $\tau_k$ is denoted by $\Theta_{a,n_k} = \text{diag} \left(\sqrt{\alpha_{u,n}^k e^{j\theta_{u,n}^k}}, \ldots, \sqrt{\alpha_{u,n}^k e^{j\theta_{u,n}^k}}\right)$, where $\alpha_{u,n}^k$ and $\theta_{u,n}^k$ respectively denote the amplification amplitude and the phase shift of the $n$-th element in the UL phase, $\forall k \in \{r, t\}$.

Let $s_{u,k}$ denote the information-carrying signal of $U_{p,k}$ with unit power, and the transmitted signal at $U_{p,k}$ is denoted by $p_{u,k} s_{u,k}$, where $p_{u,k}$ is the transmit power at $U_{p,k}$. Then, the received signal at the HAP from $U_{p,k}$ is
$$y_{h,k} = g_{u,k}^H \Theta_{a,n_k} (g_{p,k} s_{u,k} + n_r) + n_h, \quad (10)$$
where $n_h \sim \mathcal{CN} \left(0, \sigma^2_h\right)$ is the AWGN at the HAP. Due to the energy causality, we have the following constraint $\tau_k (P_{u,k} + P_{c,n}) \leq E_{h,k}$, where $P_{c,n}$ is the circuit power consumption of each WPD.

Different from the DL phase, the amplification power constraint at the aSTAR-RIS with the TS protocol during $\tau_k$ is given by
$$p_{u,k} \|\Theta_{a,n_k} g_{p,k}\|^2 + \sigma^2 \|\Theta_{a,n_k}\|^2 \leq P_a. \quad (11)$$

Thus, the achievable sum-rate at the HAP in the UL phase is expressed as
$$R = \tau_r \log \left(1 + \gamma_{u,r}\right) + \tau_t \log \left(1 + \gamma_{u,t}\right), \quad (12)$$
where $\gamma_{u,k} = \frac{p_{u,k} \|g_{u,k}^H \Theta_{a,n_k} g_{p,k}\|^2}{\sigma^2 \|g_{u,k}^H \Theta_{a,n_k}\|^2 + \sigma^2_{u,k}}$ is the SNR of $U_{p,k}$.

**III. UPLINK SUM-RATE MAXIMIZATION**

In this section, we aim to maximize the uplink sum-rate via the joint optimization of the transmission and reflection beamforming at the aSTAR-RIS, the transmit power of the HAP, the power allocation of the WPDs, and the network time scheduling. Moreover, the QoS requirements at the IRDs are considered for guarantying the DL transmission
The optimization problem is formulated as
\[
\Theta_{d,k} \max_{p_{d,k}, p_{u,k}} R
\]  
\[
s.t. \quad (9), (11),  
R_{i,k} \geq R_{i,k,\min}, \forall k,  
|H_{i,r}^H(\Theta_{d,r}, h)|^2 \geq |H_{d,j}^H(\Theta_{d,j}, h)|^2,  
\tau_k (p_{u,k} + P_{c,u}) \leq E_{h,k}, \forall k,  
p_{d,r} + p_{d,t} = P_{d}, p_{d,t} \leq \tau_d,  
\tau_d + \tau_r + \tau_t \leq 1, \tau_d \geq 0, \tau_r \geq 0, \forall k,  
\beta_{d,n}^l + \beta_{d,n}^r = 1, \alpha_{d,n} \leq \alpha_{\max}, \forall k,  
0 \leq \theta_{d,n}^l \leq 2\pi,  
p_{d,k} \geq 0, p_{u,k} \geq 0, \forall k, \]

where (14) is the QoS constraint at the IRDs and \(R_{i,k,\min}\) denotes the minimum data rate of \(U_{i,k}\), (15) is the channel condition constraint for performing the DL NOMA, (16) is the energy causality constraint at WPDs, (17) is the transmit power constraint of the HAP, (18) is the time scheduling constraint, (19) is the amplitude constraint of each element at the aSTAR-RIS, (20) is the amplification gain of each element, (21) is the phase shift constraints of each element, and (22) indicates that the power variables are nonnegative.

It is straightforward that the objective function and the constraints i.e., (9), (11), (14), and (16), are non-convex due to the coupled variables. In addition, we observe that \(\textbf{P0}\) is a quadratic fractional programming problem, which is challenging to directly obtain the optimal solution by standard optimization techniques. To solve the exceedingly non-convex problem, we propose an efficient algorithm based on the AO framework to find a near-optimal solution. In particular, we decompose the optimization problem into three sub-problems and solve them iteratively. In each iteration, we first optimize the DL aSTAR-RIS coefficients and UL transmit power optimization by using the SDR method. Then, we perform the UL aSTAR-RIS coefficients and DL transmit power optimization by using the FP and SDR methods. Finally, the time scheduling optimization is executed by using the interior-point method.

A. DL aSTAR-RIS Coefficients and UL Transmit Power Optimization

In this sub-section, we focus on optimizing the DL reflection and transmission coefficients of the aSTAR-RIS and the UL transmit power at each WPD. Specifically, we optimize \(\{\Theta_{d,k}, p_{u,k}\}\) with the fixed \(\{\Theta_{u,k}, p_{d,k}, \tau\}\). The sub-problem can be formulated as follows
\[
\textbf{P1} \quad \max_{\Theta_{d,k}, p_{u,k}} R
\]
\[
s.t. \quad (9), (11), (14) - (16),  
\beta_{d,n}^l + \beta_{d,n}^r = 1, \quad \alpha_{d,n} \leq \alpha_{\max},  
0 \leq \theta_{d,n}^l \leq 2\pi, \quad p_{u,k} \geq 0, \quad \forall k \in \{r, t\}. \]

However, \(\textbf{P1}\) is still highly non-convex. To address this issue, we introduce some auxiliary variables to reformulate it. Let \(V_{d,k} = v_{d,k}v_{d,k}^H\), where \(v_{d,k} = [\phi_{d,k,1}, \ldots, \phi_{d,k,N}]^H \in \mathbb{C}^{N \times 1}\) and \(\phi_{d,k,n} = \sqrt{\alpha_{d,n}} \beta_{d,n}^l e^{j\theta_{d,n}^l}\). Clearly, the matrix \(V_{d,k}\) is rank one and positive semidefinite. Derived from the constraints in (24), \(V_{d,k}\) satisfies the following constraints
\[
V_{d,k} \geq 0, \quad \text{rank}(V_{d,k}) = 1,  
[V_{d,l}]_{n,n} \leq \alpha_{\max}, \quad \forall l \in \{r, t\}. \]

Then, let \(H = \text{diag}(|h_1|^2, \ldots, |h_N|^2)\), \(H_{i,k} = \text{diag}(h_{i,k})\) and \(Q_{i,k} = H_{i,k}^H H_{i,k}, \forall l \in \{i, p\}, \forall k \in \{r, t\}\). In addition, let \(T_{in,k} = P_d Tr[V_{d,k}Q_{p,k}] + \sigma_r^2 Tr[V_{d,k}H_{i,k}^HH_{i,k}]\), \(\chi_{i,k} = \sigma_r^2 Tr[V_{d,k}H_{i,k}^HH_{i,k}] + \sigma_{i,k}^2\), and \(c_{i,k} = 2\frac{\beta_{i,k,\min}}{\beta_{i,k,\min} - 1}\). Based on these new auxiliary variables, the constraints (9), (14), (15), and (16) can be recast as
\[
\sum_k P_d Tr[V_{d,k}H] + \sigma_r^2 Tr[V_{d,k}] \leq P_a,  
p_{d,r} Tr[V_{d,r}Q_{i,r}] \geq c_{i,r} \chi_{i,r},  
p_{d,t} Tr[V_{d,k}Q_{i,k}] \geq c_{i,t} (p_{d,r} Tr[V_{d,k}Q_{i,k}] + \chi_{i,k}),  
Tr[V_{d,r}Q_{i,r}] \geq Tr[V_{d,k}Q_{i,k}],  
\tau_k (p_{u,k} + P_{c,u}) \leq \tau_d \min \{q T_{in,k}, P_{sat}\}. \]

Thus, \(\textbf{P1}\) can be recast as
\[
\textbf{P2} \quad \max_{V_{d,k}, p_{u,k}} R
\]
\[
s.t. \quad (11), (26) - (32). \]

However, \(\textbf{P2}\) is still intractable because the rank-one constraint in (26) is non-convex. Therefore, we employ the SDR technique to relax it and transform \(\textbf{P2}\) into a convex semidefinite programming, which can be optimally solved with the interior-point method [25]. Similar to [26], we can prove that the obtained solution \(V_{d,k}\) by solving the relaxed version of \(\textbf{P2}\) is rank-one. That is to say, the tightness of the obtained solution is guaranteed. Hence, we then employ Singular Value Decomposition (SVD) to obtain \(v_{d,k}\) from \(V_{d,k}\). Subsequently, the optimal reflection and transmission coefficients of the aSTAR-RIS can be obtained by \(\Theta_{d,k} = \text{diag}(v_{d,k}^*)\).

B. UL aSTAR-RIS Coefficients and DL Transmit Power Optimization

Then, we optimize the UL aSTAR-RIS coefficients and DL transmit power at the HAP with the fixed \(\{\Theta_{d,k}, p_{u,k}\}\). The sub-problem is formulated as
\[
\textbf{P3} \quad \max_{\Theta_{u,k}, p_{d,k}} R
\]
\[
s.t. \quad (11), (14), (17),  
\alpha_{u,n} \leq \alpha_{\max}, \quad 0 \leq \theta_{u,n}^l \leq 2\pi, \quad p_{d,k} \geq 0. \]

We can observe that \(\textbf{P3}\) can be decomposed into two independent sub-problems, each of which maximizes \(\gamma_k\), \(\forall k \in \{r, t\}\). Let \(V_{u,k} = v_{u,k}v_{u,k}^H\), where \(v_{u,k} = \ldots\)
\[\phi_{u,k,1}, \ldots, \phi_{u,k,N}]^H \in \mathbb{C}^{N \times 1}\] and \(\phi_{u,k,n} = \sqrt{\alpha_n^k} e^{j\theta_n^k}\).

Thus, the constraints of \(V_{u,k}\) are given by
\[
V_{u,k} \succeq 0, \ \text{rank}(V_{u,k}) = 1, \quad [V_{u,k}]_{n,n} \leq \alpha_{\text{max}}.
\]

Then, the objective function can be rewritten as
\[
\gamma_k = \frac{p_{u,k} \text{Tr}(V_{u,k}Q_{u,k})}{\sigma_n^2\text{Tr}(V_{u,k}G^H G) + \sigma_n^2},
\]

where \(G = \text{diag}(g)\), and \(Q_{u,k} = G^H g_{p,k}g_{p,k}^H G\). It is a quadratic fractional programming problem since the employment of the aSTAR-RIS introduces an additional amplified noise term in the denominator. In addition, the aSTAR-RIS power constraints in (11) can be reformulated as
\[
p_{u,k}\text{Tr}(V_{u,k}G_{p,k}) + \sigma_n^2\text{Tr}(V_{u,k}) \leq P_a, \quad (39)
\]

As a result, the subproblem of maximizing \(\gamma_k\) with respect to \(\Theta_{u,k}\) is formulated as
\[
\begin{align*}
\text{(P4)} & \quad \max_{V_{u,k},p_{d,k}} \gamma_k \\
& \quad \text{s.t.} \quad (14), (17), (36), (37), (39).
\end{align*}
\]

It is obvious that P4 is also non-convex. Based on the nonlinear fractional programming theory proposed in [27], we transform the fractional objective function into a subtractive form. Denote \(\gamma_k^*\) as the maximum SNR of \(U_{p,k}\), and \(\gamma_k^* = \max_{V_{u,k},p_{d,k}} F_{\text{num}}/F_{\text{denom}}\), where \(F_{\text{num}} = p_{u,k}\text{Tr}(V_{u,k}Q_{u,k})\), \(F_{\text{denom}} = \sigma_n^2\text{Tr}(V_{u,k}G^H G) + \sigma_n^2\).

Then, the maximum SNR can be achieved if and only if
\[
\max_{V_{u,k},p_{d,k}} (F_{\text{num}} - \gamma_k^* F_{\text{denom}}) = 0.
\]

The sub-problem is thus given by
\[
\begin{align*}
\text{(P5)} & \quad \max_{V_{u,k},p_{d,k}} F_{\text{num}} - \gamma_k^* F_{\text{denom}} \\
& \quad \text{s.t.} \quad (14), (17), (36), (37), (39),
\end{align*}
\]

where \(\gamma_k^* = F_{\text{num}}/F_{\text{denom}}, F_{\text{num}}\) and \(F_{\text{denom}}\) are the optimal function at the last iteration. This problem can be solved by the iterative algorithm proposed in [27]. At the beginning of the algorithm, \(\gamma_k^*\) is initialized as 0. In each iteration, we also apply the SDR method to solve P5 with the result \(\gamma_k^*\) derived in the previous iteration and update it again in this round. The optimal solution denoted by \(V_{u,k}^*\) and \(p_{d,k}^*\) can be derived until the algorithm is converged. Similar to the relaxed problem P2, \(V_{u,k}^*\) satisfies the rank-one constraint, and \(\{\Theta_{u,k}^*, p_{d,k}^*\}\) can be finally derived.

### C. Time Scheduling Optimization

We proceed to optimize time scheduling and the transmit power at each WPD. Let \(e_{u,k} = \frac{\|g^u_{p,k}\|^2}{\sigma_n^2\|g^u_{\Theta_{u,k}}\|^2 + \sigma_n^2}\) and \(e_{u,k} = p_{u,k}\tau_k\). The corresponding sub-problem is given by
\[
\begin{align*}
\text{(P6)} & \quad \max_{T_{e,u},k} R \sum_k \tau_k \log \left(1 + \frac{e_{u,k} e_{p,k}}{\tau_k} \right) \\
& \quad \text{s.t.} \quad (14), (18), \\
& \quad e_{u,k} + \tau_k P_{e,u} \leq E_{h,k}, \\
& \quad e_{u,k} \|\Theta_{u,k}g_{p,k}\|^2 + \tau_k \sigma_n^2 \|\Theta_{u,k}\|^2 \leq \tau_k P_a.
\end{align*}
\]

It can be easily verified that P6 is a convex optimization problem that can be solved by the interior-point method. Thus, we obtain the optimal time scheduling solutions, denoted by \(\tau^*\). Then, the optimal transmit power at each WPD is given by \(p_{u,k}^* = e_{u,k}^*/\tau_k^*\).

The proposed AO algorithm for solving the uplink sum-rate maximization Optimization problem is summarized in Algorithm 1. The convergence of Algorithm 1 is guaranteed as the objective function of (13) increases after each step and is bound from above.

#### Algorithm 1 AO Algorithm for Solving Uplink Sum-rate Maximization Problem

1. **Initialization**: \(\Theta_{d,k}, \Theta_{u,k}, P_{d,k}, P_{u,k}, \tau, \forall k \in \{r, t\}.
2. **repeat**
3. **Given** \{\(\Theta_{d,k}, P_{d,k}, \tau\), update \(\Theta_{d,k}, P_{d,k}\) by solving P2 via using the SDR method.
4. **Given** \(\Theta_{d,k}, P_{d,k}, \tau\), update \(\{\Theta_{u,k}, P_{d,k}\}\) by solving P5 via using the FP and SDR methods.
5. **Given** \(\Theta_{d,k}, \Theta_{u,k}, P_{d,k}\), update \(\{\tau, P_{d,k}\}\) by solving P6 via using the interior-point method.
6. **until** \(R\) converged.
7. **return** \(\Theta_{u,k}, \Theta_{d,k}, P_{d,k}, P_{u,k}, \tau\).

### IV. Numerical Results

In this section, numerical results are provided to evaluate the effectiveness of the proposed scheme. We consider a two-dimensional Cartesian coordinate to simulate the system deployment, where the HAP and the aSTAR-RIS are respectively deployed at (0, 0) and \((x_r, 0)\), and \(U_{i,r}\) and \(U_{i,t}\) are randomly located on a half-annulus centered at \((x_r, 0)\) of 2 m to 3 m, and that of 4 m to 5 m, respectively. Similar to [15], we consider that the channels follow large-scale fading and small-scale fading. For the aSTAR-RIS related links, the path-loss at the reference distance of 1 m is set to \(\lambda = -30\) dB and the path-loss exponent is set to 2.2. We assume that the aSTAR-RIS related channels follow the Rician fading, and the Rician factor is set to 3 dB. Unless otherwise stated, other parameters are given as follows: \(\eta = 0.8\), \(P_{c,u} = 5\) \(\mu W\), \(P_{\text{sat}} = 50\) mW, \(\sigma_r^2 = \sigma_h^2 = \sigma_t^2 = -90\) dBm, \(P_d = 30\) dBm, \(P_a = 30\) dBm, \(a_{\text{max}} = 25\) dB, \(N = 32, x_r = 8\) m. To guarantee the fairness of IRDs, the minimum date rate requirements of \(U_{i,r}\) and \(U_{i,t}\) are set as 2 bps/Hz and 0.2 bps/Hz, respectively.

For comparison, we also evaluate the performance of the following benchmark schemes:

1. **Active STAR-RIS-aided system with fixed phase shifts scheme (aSTAR-RIS-FPS)**: Setting all phase shifts to...
0, and only optimizing the amplitude and amplification coefficients at the aSTAR-RIS.

2) Passive STAR-RIS-aided system scheme (pSTAR-RIS):
Setting the number of elements of the passive STAR-RIS to 4N.

In Fig. 2, we investigate the convergence performance of the proposed AO algorithm by showing the variation of sum-rate over the iterations. It is observed that the sum-rate finally converges to a stable value after nearly 6 iterations, which demonstrates that the proposed AO algorithm with different parameter settings can converge quickly.

In Fig. 3, the impact of the transmit power at the HAP on the sum-rate is investigated. It is clear that the sum-rate increases as the transmit power increases. Since the WPDs can harvest more energy from the HAP with higher transmit power, more harvested energy can be used to provide transmit power and transmit time at the WPDs in the UL. Moreover, the sum-rate for aSTAR-RIS schemes tends to saturate when $P_d \geq 40$ dBm. Based on the amplification power constraint of the aSTAR-RIS shown in (9), as the transmit power increases, the constraint becomes active. Therefore, the harvested energy at the WPDs assisted by the aSTAR-RIS is limited, which restricts the achievable uplink sum-rate. This result indicates the importance of designing the amplification gain of the aSTAR-RIS according to the transmit power at the HAP. Furthermore, we can find that the proposed scheme always achieves the highest sum-rate. In particular, compared with the pSTAR-RIS scheme equipped with 128 elements, our proposed aSTAR-RIS scheme equipped with 32 elements can achieve performance improvements of up to 1721.2% when $P_d = 20$ dBm. The main reason is that the aSTAR-RIS has the ability to amplify incident signals in both the DL SWIPT and UL WIT phases.

Fig. 4 shows the relationship between the sum-rate and the number of elements at the STAR-RIS. The range of $N$ for the aSTAR-RIS is 8 to 56, while the number of elements for the pSTAR-RIS is four times that of the aSTAR-RIS. With an increasing number of elements, the sum-rate increases due to the fact that more transmission links can be provided between the HAP and the WPDs. Moreover, it can also be seen that the proposed scheme remarkably outperforms the benchmark schemes. Notably, the performance gap between the proposed scheme and the aSTAR-RIS-FPS scheme grows as the number of elements increases. Since the aSTAR-RIS-FPS scheme adopts fixed phase shifts, the aSTAR-RIS cannot adaptively adjust its beamforming coefficients based on different channels. It also can be concluded that the passive STAR-RIS without amplification needs a large number of elements to bridge the performance gap with the aSTAR-RIS.

Fig. 5 indicates the effect of the distance between the HAP and the STAR-RIS on the sum-rate. The sum-rate decreases with increasing distance because the severe path loss increases sharply. Consequently, the efficiency of WIT and WET has degraded with longer distances, so the system needs more DL time to satisfy the QoS requirements of IRDs and the EH requirements of WPDs. In terms of the pSTAR-RIS scheme, the performance drop is more pronounced due to the “double fading” effect. Again, the performance of the aSTAR-RIS schemes is superior to that of the pSTAR-RIS scheme, which demonstrates that the aSTAR-RIS can effectively mitigate the “double fading” effect.
V. CONCLUSIONS

This paper has studied an aSTAR-RIS assisted WIPT system, which consists of the DL SWIPT from the HAP to the IRDs and the WPDs and the UL WIIT from the WPDs to the HAP. We have formulated an uplink sum-rate maximization problem while considering the QoS requirements of IRDs. For this purpose, we jointly optimized the transmit power of the HAP, the power allocation of the WPDs, the time scheduling, and the transmission and reflection beamforming at the aSTAR-RIS. Since the formulated problem is non-convex, an alternating optimization algorithm was proposed based on SDR and FP techniques to solve it constructively. A comparison between the passive STAR-RIS and aSTAR-RIS assisted WIPT systems has been provided by the numerical results. Compared to the passive STAR-RIS with 128 elements scheme, the proposed aSTAR-RIS with 32 elements scheme can achieve up to 640.68% performance gains in our settings. In conclusion, these results confirmed the advantages of integrating the aSTAR-RIS into WIPT systems.

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under and Grant 62071242, in part by the Key Program of Marine Economy Development Special Foundation of Department of Natural Resources of Guangdong Province under Grant GDNRC[2023]24, in part by the Postgraduate Research & Practice Innovation Program of Jiangsu Province under Grant KYCX22_0967, KYCX21_0744 and SJCX23_0254.

REFERENCES