Joint Beamforming Design for Cooperative Double-RIS Aided mmWave Multi-User MIMO Communications

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Abstract—To alleviate the blockage effect and explore the potential of reconﬁgurable intelligent surface (RIS) assisted communication, we investigate the cooperative double-RIS assisted multi-user millimeter wave (mmWave) multiple-input multiple-output (MIMO) communications. To improve system performance, we jointly optimize the transmit beamforming matrix at the base station and the phase shift matrices at RISs to maximize the system sum rate, which is an intractable non-convex problem. To solve the problem, we propose an efﬁcient alternating optimization algorithm based on the techniques of weighted minimum mean square error (WMMSE), Lagrange multiplier and majorization-minimization (MM). Simulation results validate the effectiveness of the proposed algorithm, as well as the superiority of double-RIS in improving system performance.

Index Terms—reconﬁgurable intelligent surface (RIS); millimeter wave (mmWave); MIMO; beamforming.

I. INTRODUCTION

Millimeter wave (mmWave) communications have been considered as one of the most promising technologies to meet the exponentially increasing demand on mobile data traffic due to the enormous bandwidth available in the mmWave band. However, the high directivity makes mmWave signals more sensitive to blockages. Recently, reconﬁgurable intelligent surface (RIS) has emerged as a promising technology to overcome the blockage problem of mmWave communication systems with low-cost and low-power consumption [1].

In practical, deploying multiple RISs is usually recommended to provide more spatial freedom and additional paths to combat severe path loss and overcome blockage problem. For example, in [2], the sum rate maximization problem in multi-RIS-aided multi-user multiple-input single-output (MISO) mmWave systems is studied. In [3], the authors proposed an alternating optimization (AO) algorithm to maximize the system energy efﬁciency. The secure beamforming problem is investigated via successive convex approximation method in [4]. However, most of these studies focused on the scenarios of single-RIS or multiple distributed RISs without considering the beneﬁts of collaboration among multiple RISs.

To further exploit the potential of RIS-aided communications, some studies have studied double-RIS cooperatively aided communications [5]–[8]. To be speciﬁc, a double-RIS assisted single-input single-output (SISO) system under the line-of-sight (LoS) channels is studied in [5]. Following this, [6] and [7] extended double-RIS to multi-user MISO systems. In [6], the problem of maximizing the minimum signal interference to noise ratio (SINR) is studied and the energy efﬁciency maximization problem is investigated in [7]. To maximize the weighted sum rate of a double-RIS-aided multi-user mmWave system, a block coordinate descent (BCD) algorithm is proposed in [8]. The results of these studies show that collaborative double-RIS has great potential to improve system performance compared to traditional single-RIS or multiple distributed RISs.

Although these studies have investigated the performance of double-RIS assisted communications, they mostly focused on single-antenna user scenarios. For multi-antenna user scenarios, especially multi-user scenarios, the joint active and passive beamforming design has not been fully investigated yet. For double-RIS aided multi-user mmWave MIMO communications, the joint active and passive beamforming design is much more challenging and complex due to its complex objective function as well as the non-convex variables coupled in it. In addition, most studies use CVX toolbox to solve optimization problem simply, and the computational complexity is relatively high, which is not applicable in double-RIS assisted multi-user MIMO systems. Therefore, it is necessary to design an effective joint beamforming algorithm.

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II. SYSTEM MODEL AND PROBLEM FORMULATION

As shown in Fig. 1, a cooperative double-RIS aided multi-user mmWave MIMO system is considered, which consists of one base station (BS), two RISs and \( K \) users. We assume that the direct links between the BS and users are blocked by obstacles. To assist the downlink communication, two cooperative RISs (marked as RIS 1 and RIS 2) are deployed near the BS and users, respectively. By adjusting the phase shifts of RIS reflecting elements, virtual LoS links can be established between the BS and users. Therefore, the quality of service for users can be guaranteed. Besides, we assume that the BS is equipped with \( N_t \) transmit antennas and each user is equipped with \( N_r \) receive antennas. Consider a total number of \( M \) reflecting elements for two RISs, where RIS 1 and RIS 2 consist of \( M_1 \) and \( M_2 \) reflecting elements, respectively, with \( M_1 + M_2 = M \). For clear denotation, we use \( K = \{1, 2, \ldots, K\} \) to denote the set of all users.

Let \( G_1 \in \mathbb{C}^{M_1 \times N_r} \), \( G_2 \in \mathbb{C}^{M_2 \times N_r} \), \( F \in \mathbb{C}^{M_2 \times M_1} \), \( H_{1,k} \in \mathbb{C}^{N_r \times M_1} \) and \( H_{2,k} \in \mathbb{C}^{N_r \times M_2} \) denote the channels for the BS to RIS 1, the BS to RIS 2, the RIS 1 to RIS 2, the RIS 1 to user \( k \), and the RIS 2 to user \( k \), respectively. Let \( \Theta_i = \text{diag} (\theta_{1,i}, \ldots, \theta_{M_i,i}) \) denote the phase shift matrix of RIS \( i \), \( i = 1, 2 \) and \( \theta_{m,i} = e^{j \theta_m,i} \in [0, 2\pi) \) represents the phase shift of the \( m \)-th reflecting element of RIS \( i \). Note that the BS-RIS 2-RIS 1-user \( k \) link is ignored due to the longer link distance. Then, based on the above representation, the equivalent cascade channel between the BS and user \( k \) can be expressed as

\[
H_k = H_{1,k} \Theta_1 G_1 + H_{2,k} \Theta_2 G_2 + H_{2,k} \Theta_2 \Theta_1 G_1. 
\]

We suppose that the BS uses a uniform planar array (UPA) and the user uses a uniform line array (ULA). We use a geometric channel model to characterize mmWave channels. Hence, the BS-RIS 1 channel \( G_1 \) can be expressed as

\[
G_1 = \sum_{l=1}^{N_p} \rho_l a_r (\varphi_l^1, \varphi_l^0) a_t^H (\psi_l^0, \psi_l^1),
\]

where \( N_p \) denotes the total number of paths, \( \rho_l \) indicates the complex gain of the \( l \)-th path, \( a_r \) and \( a_t \) are the array response vectors at the BS and RIS 1, respectively, \( \varphi_l^1 (\varphi_l^0) \) and \( \psi_l^1 (\psi_l^0) \) represent the azimuth and (elevation) angles of arrivals and departures, respectively. Then, the RIS 1-user \( k \) channel \( H_{1,k} \) is given by

\[
H_{1,k} = \sum_{l=1}^{N_p} \zeta_l a_r (\vartheta_l) a_t^H (\omega_l^0, \omega_l^1),
\]

where \( \zeta_l \) is the complex gain of the \( l \)-th path and \( \vartheta_l \) is the corresponding angle of arrival from RIS 1 to user \( k \).

For a ULA, the array response vector \( a_r (\vartheta_l) \) can be expressed as

\[
a_r (\vartheta_l) = \begin{bmatrix} 1, e^{j \pi \sin(\vartheta_l)}, \ldots, e^{j \pi (N_r - 1) \sin(\vartheta_l)} \end{bmatrix}^T.
\]

For a UPA, the array response vector \( a_r (\psi, \varphi) \) is given by

\[
a_r (\psi, \varphi) = \begin{bmatrix} 1, e^{j \pi (m \sin(\psi) \sin(\varphi) + n \cos(\psi))}, \ldots, e^{j \pi ((N_r - 1) \sin(\psi) \sin(\varphi) + (N_p - 1) \cos(\psi))} \end{bmatrix}^T,
\]

where \( N_t = N_x \times N_y \), \( N_x \) and \( N_y \) denote the horizontal and vertical antenna element, respectively. Other channels and array response vectors can be similarly defined.

Let \( x = \sum_{k \in K} W_k s_k \) denote the transmitted signal at the BS, where \( s_k \in \mathbb{C}^{N_s \times 1} \) is the transmit symbol for user \( k \), \( N_s \) is the number of information streams, \( W_k \in \mathbb{C}^{N_r \times N_s} \) is the corresponding transmit beamforming matrix. For the transmit symbol \( s_k \), we assume that \( \mathbb{E} [s_k s_k^H] = I \) and \( \mathbb{E} [s_k s_{j,k}^H] = 0 \) for any \( k \neq j \). Hence, the received signal at user \( k \) is

\[
y_k = \bar{H}_k W_k s_k + \bar{H}_k \sum_{j \in K, j \neq k} W_j s_j + n_k,
\]

where \( n_k \sim \mathcal{CN} (0, \sigma_n^2 I) \) is the received additive white Gaussian noise with zero mean and variance \( \sigma_n^2 \) at user \( k \). Then, the achievable rate for user \( k \) is given by

\[
R_k = \log \det \left( \mathbf{I} + W_k^H \bar{H}_k^H C_k^{-1} H_k W_k \right),
\]

where \( C_k = \bar{H}_k \sum_{j \in K, j \neq k} W_j W_j^H \bar{H}_j^H + \sigma_n^2 \mathbf{I} \) is the covariance matrix of the interference and noise at user \( k \).

We aim to maximize the system sum-rate by jointly optimizing the transmit beamforming matrix at the BS and the phase shift matrices at RISs. Therefore, the problem can be formulated as
\[
\max_{W_k, \Theta_1, \Theta_2} f(W_k, \Theta_1, \Theta_2) = \sum_{k \in \mathcal{K}} R_k
\] (8)
\[
s.t. \sum_{k \in \mathcal{K}} \text{Tr}(W_k W_k^H) \leq P_{\text{max}}, \quad (8a)
\]
\[
|\theta_{i,m}| = 1, \forall m = 1, \ldots, M_i, i = 1, 2, \quad (8b)
\]
where constraint (8a) indicates the transmit power constraint of the BS, \( P_{\text{max}} \) is the maximum transmit power, and constraint (8b) is the phase shift constraint for RISs. The formulated problem is non-convex and intractable due to the non-convex objective function and constraint (8b).

### III. Proposed Joint Beamforming Algorithm

In this section, we first transform the formulated problem into an equivalent WMMSE minimization problem. Next, we use the Lagrange multiplier method to design the transmit beamforming matrix and derive the closed expression. For the phase shift matrix of RIS, we propose a MM-based algorithm that can update the phase shift matrix of RIS with a closed-form expression in each iteration.

#### A. Problem Transformation

The sum rate maximization problem in (8) can be equivalently transformed into a WMMSE problem. At the user side, user \( k \) recovers its desired signal \( s_k \) via a linear decoding matrix \( V_k \in \mathbb{C}^{N_r \times N_r} \), so the estimated signal of user \( k \) is given by \( \hat{s}_k = V_k y_k \). Hence, the corresponding MSE matrix can be written as

\[
E_k = \mathbb{E} \left[ (\hat{s}_k - s_k)(\hat{s}_k - s_k)^H \right] = V_k \bar{H}_k Q \bar{H}_k^H V_k^H - V_k \bar{H}_k W_k \bar{H}_k^H V_k^H \leq P_{\text{max}}.
\] (9)

Then, the optimal \( V_k \) can be obtained by setting \( \frac{\partial E_k}{\partial V_k} \) to zero, which can be expressed as

\[
V_k = W_k^H \bar{H}_k^H (\bar{H}_k Q \bar{H}_k^H + \sigma_k^2 I)^{-1}.
\] (10)

When \( V_k \) is fixed, substituting (10) into (9), the \( E_k \) can be rewritten as

\[
E_k = I - W_k^H \bar{H}_k^H J_k^{-1} \bar{H}_k W_k,
\] (11)

where \( J_k = \bar{H}_k Q \bar{H}_k^H + \sigma_k^2 I \).

The original optimization problem in (8) is equivalent to the following WMMSE minimization problem by introducing a auxiliary matrix \( Z_k \geq 0, \forall k \in \mathcal{K} \) [9], which can be written as

\[
\max_{W_k, Z_k, \Theta_1, \Theta_2} f_2(W_k, Z_k, \Theta_1, \Theta_2)
\] (12)
\[
s.t. (8a), (8b), \quad (12a)
\]
\[
Z_k \geq 0, \forall k \in \mathcal{K}, \quad (12b)
\]

where

\[
f_2(W_k, Z_k, \Theta_1, \Theta_2) = \sum_{k \in \mathcal{K}} \log \det (Z_k) - \text{Tr} (Z_k E_k). \quad (13)
\]

With fixed other variables, the optimal \( Z_k \) can be obtained by checking the first order optimality condition, then we have \( Z_k = E_k^{-1} \).

#### B. Transmit Beamforming Optimization

In this subsection, we focus on optimizing the transmit beamforming matrix \( W_k \) with fixed \( Z_k, \Theta_1 \) and \( \Theta_2 \). Substituting (9) into (12), the optimization problem for \( W_k \) is given by

\[
\min_{W_k} f_4(W_k)
\] (14)
\[
s.t. \sum_{k \in \mathcal{K}} \text{Tr}(W_k W_k^H) \leq P_{\text{max}} \quad (14a)
\]

where

\[
f_4(W_k) = \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{K}} \sum_{j \in \mathcal{K}} \text{Tr}(Z_k V_j H_k W_k^H H_k^H V_j^H)
\]
\[
- \sum_{k \in \mathcal{K}} \text{Tr}(Z_k V_k H_k W_k)
\]
\[
- \sum_{k \in \mathcal{K}} \text{Tr}(Z_k W_k^H H_k^H V_k^H).
\] (15)

Obviously, it is a convex quadratically constrained quadratic programming (QCQP) problem, which can be solved directly by using convex optimization toolbox CVX. In order to obtain the closed form solution, the Lagrange multipliers method is adopted to cope with the transmit power constraint by introducing a dual variable \( \lambda \geq 0 \). Then, the Lagrange function is given as follows

\[
\mathcal{L}(W_k, \lambda) = \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{K}} \sum_{j \in \mathcal{K}} \text{Tr}(Z_k V_j H_k W_j^H H_k^H V_j^H)
\]
\[
- \sum_{k \in \mathcal{K}} \text{Tr}(Z_k V_k H_k W_k)
\]
\[
- \sum_{k \in \mathcal{K}} \text{Tr}(Z_k W_k^H H_k^H V_k^H) + \lambda \left( \sum_{k \in \mathcal{K}} \text{Tr}(W_k W_k^H) - P_{\text{max}} \right).
\] (16)

Then, the optimal \( W_k \) can be obtained by setting the first-order derivative to zero, i.e., \( \frac{\partial \mathcal{L}(W_k, \lambda)}{\partial W_k} = 0 \). Thus, the optimal \( W_k \) can be expressed as

\[
W_k = \left( \sum_{j \in \mathcal{K}} H_j^H V_j Z_j V_j + \lambda I \right)^{-1} H_k^H V_k Z_k,
\] (17)

where the optimal \( \lambda \) should be chosen to satisfy the following equation

\[
\lambda = \min \left\{ \lambda \geq 0 : \sum_{k \in \mathcal{K}} \text{Tr}(W_k W_k^H) = P_{\text{max}} \right\},
\] (18)

which can be found by the bisection method.
C. RIS Phase Shift Optimization

Here, with fixed transmit beamforming matrix $W_k$, the optimization of the phase shift at RIS is discussed. Particularly, the corresponding optimization problem can be written as

$$
\begin{align*}
\min_{\Theta_1, \Theta_2} & \quad f_5(\Theta_1, \Theta_2) \\
\text{s.t.} & \quad |\theta_{i,m}| = 1, \forall m = 1, ..., M_i, i = 1, 2,
\end{align*}
$$

where the objective function $f_5(\Theta_1, \Theta_2)$ is given as

$$
\begin{align*}
f_5(\Theta_1, \Theta_2) = & \sum_{k \in \mathcal{K}} \text{Tr} \left( Z_k V_k \tilde{H}_k Q H_k^H V_k^H \right) - \sum_{k \in \mathcal{K}} \text{Tr} \left( Z_k V_k \tilde{H}_k W_k \right) - \sum_{k \in \mathcal{K}} \text{Tr} \left( Z_k W_k^H \tilde{H}_k^H V_k^H \right).
\end{align*}
$$

(19a)

We note that $\Theta_1$ and $\Theta_2$ are symmetric in $\tilde{H}_k$. Thus, the proposed optimization method for $\Theta_1$ is suitable for $\Theta_2$. To this end, we focus on the optimization of $\Theta_1$ with fixed $\Theta_2$.

Firstly, with fixed $W_k$ and $\Theta_2$, we have the following proposition.

Proposition 1. With fixed $W_k$ and $\Theta_2$, the optimization problem for $\Theta_1$ can be transformed into

$$
\begin{align*}
\min_{\Theta_1} & \quad f_{6a}(\Theta_1) \\
\text{s.t.} & \quad |\theta_{1,m}| = 1, \forall m = 1, ..., M_1,
\end{align*}
$$

where

$$
\begin{align*}
f_{6a}(\Theta_1) = & \text{Tr} \left( \Theta_1^H \Gamma \Theta_1 \Pi \right) + \text{Tr} \left( \Theta_1 D \Pi^T \right) + \text{Tr} \left( \Theta_1 D \Pi^T \right),
\end{align*}
$$

(21)

and $\Gamma$, $\Pi$, $D$ are respectively defined as

$$
\begin{align*}
\Gamma = & \sum_{k \in \mathcal{K}} A_k^H V_k^H Z_k V_k A_k, \\
\Pi = & G_1 Q G_1^H, \\
D = & \sum_{k \in \mathcal{K}} \left( A_k^H V_k^H Z_k V_k B_k Q H_k^H Z_k W_k^H G_1^H \right).
\end{align*}
$$

(23)

Proof: First of all, the equivalent cascade channel $\tilde{H}_k$ can be rewritten as

$$
\begin{align*}
\tilde{H}_k = & (H_{1,k} \Theta_1 G_1 + H_{2,k} \Theta_2 G_2 + H_{2,k} \Theta_2 F_\Theta_1 G_1) \\
= & (H_{1,k} + H_{2,k} \Theta_2 F) \Theta_1 G_1 + H_{2,k} \Theta_2 G_2
\end{align*}
$$

(24)

where $A_k = H_{1,k} + H_{2,k} \Theta_2 F$ and $B_k = H_{2,k} \Theta_2 G_2$. Then, substituting (24) into (20), the $f_5(\Theta_1, \Theta_2)$ with fixed $\Theta_2$ can be rewritten as $f_6(\Theta_1)$, which is given by (25) shown at the top of next page. Omitting items that are irrelevant to $\Theta_1$, the function $f_6(\Theta_1)$ can be further simplified and rewritten as $f_{6a}(\Theta_1)$, which is given by (26) shown at the top of next page, where $\Gamma$, $\Pi$ and $D$ are respectively given by (23). ■

According to [8], the following relationships hold

$$
\begin{align*}
\text{Tr} \left( \Theta_1^H \Gamma \Theta_1 \Pi \right) = & \theta_1^H (\Gamma \odot \Pi^T) \theta_1, \\
\text{Tr} \left( D \theta_1^H \right) = & d^H \theta_1, \\
\text{Tr} \left( \Theta_1 D \theta_1^H \right) = & \theta_1^H d,
\end{align*}
$$

(27)

where $\odot$ is the Hadamard product operator, $\theta_1 = [\theta_{1,1}, ..., \theta_{1,M_1}]^T$ and $d = [D_{1,1}, ..., D_{M_1,M_1}]^T$. Therefore, substituting (27) into (21), the optimization problem for $\Theta_1$ can be recast as

$$
\begin{align*}
\min_{\theta_1} & \quad f_{6b}(\theta_1) = \theta_1^H \Xi \theta_1 + d^H \theta_1 + \theta_1^H d \\
\text{s.t.} & \quad |\theta_{1,m}| = 1, \forall m = 1, ..., M_1,
\end{align*}
$$

(28a)

where $\Xi = \Gamma \odot \Pi^T$. Note that the problem in (28) is also non-convex due to the non-convex unit modulus constraint (28a). In the following, we propose a majorization-minimization (MM)-based algorithm for solving the problem.

The main idea of the MM algorithm is to solve a difficult problem by constructing a series of more tractable approximation subproblems, the key of which is to construct a series of convex surrogate functions to obtain the optimal solution in each round of iteration [10]. Specifically, let us assume that in the $t$-th iteration, the surrogate function of $f_{6b}(\theta_1)$ is represented as $\hat{f}_{6b}(\theta_1 | \theta_1^t)$, which needs to satisfy the following three properties

$$
\begin{align*}
\hat{f}_{6b}(\theta_1 | \theta_1^t) & \geq f_{6b}(\theta_1), \\
\hat{f}_{6b}(\theta_1 | \theta_1^t) & = f_{6b}(\theta_1^t), \\
\nabla_{\theta_1} \hat{f}_{6b}(\theta_1 | \theta_1^t) & = \nabla_{\theta_1} f_{6b}(\theta_1^t).
\end{align*}
$$

(29)

These properties indicate that the constructed surrogate function $\hat{f}_{6b}(\theta_1 | \theta_1^t)$ should be the upper bound of the original function $f_{6b}(\theta_1)$, and that at fixed point $\theta_1 = \theta_1^t$, both of them have the same value and gradient.

According to [11], for any given $\theta_1^t$, we have

$$
\begin{align*}
\theta_1^H \Xi \theta_1 & \leq \mu \theta_1^H \theta_1 - 2 \Re \left( \theta_1^H (\mu I - \Xi) \theta_1^t \right) + \theta_1^H (\mu I - \Xi) \theta_1^t, \\
& = \theta_1^H \Xi \theta_1 + 2 \Re (\theta_1^H d).
\end{align*}
$$

(30)

where $\mu$ is the maximum eigenvalue of $\Xi$.

To satisfy the three properties in (29), according to the inequality in (30), we consider constructing the surrogate function $\hat{f}_{6b}(\theta_1 | \theta_1^t)$ at fixed point $\theta_1 = \theta_1^t$ as follows

$$
\begin{align*}
\hat{f}_{6b}(\theta_1 | \theta_1^t) = & \mu \theta_1^H \theta_1 - 2 \Re (\theta_1^H (\mu I - \Xi) \theta_1^t) + (\theta_1^t)^H (\mu I - \Xi) \theta_1^t + 2 \Re (\theta_1^H d).
\end{align*}
$$

(31)

Since $\theta_1^H \theta_1 = M_1$, we have $\mu \theta_1^H \theta_1 = \mu M_1$, which is a constant. And the third item is irrelevant to $\theta_1$, which can be omitted when optimizing $\theta_1$.

To this end, we can transform the problem in (28) into solving the following problem

$$
\begin{align*}
\max_{\theta_1} & \quad 2 \Re (\theta_1^H q') \\
\text{s.t.} & \quad |\theta_{1,m}| = 1, \forall m = 1, ..., M_1,
\end{align*}
$$

(32a)

where $q' = (\mu I - \Xi) \theta_1^t - d$. Therefore, the optimal solution of $\theta_1$ at the $(t + 1)$-th iteration can be obtained by

$$
\begin{align*}
\theta_1^{t+1} = & e^{j \arg(q')}.
\end{align*}
$$

(33)
the optimal computational complexity is $O(a_{\lambda}G_1a_1^Hv_k^H) - \sum_{k \in K} (Z_kV_k(a_k\Theta_1G_1 + B_k)W_k) = \sum_{k \in K} \left\{ \text{Tr} \left( Z_kV_k(a_k\Theta_1G_1 + B_k)W_k \right) \right\}
\]

Algorithm 1: The Joint Beamforming Design Based on Alternating Optimization.

1: Initialize feasible $\{W_k\}_{k \in K}$, $\Theta_1$ and $\Theta_2$;
2: repeat
3: \hspace{0.5cm} Transmit Beamforming Design:
4: \hspace{1cm} repeat
5: \hspace{1.5cm} Update $\{V_k\}_{k \in K}$ by (10);
6: \hspace{1.5cm} Update $\{E_k\}_{k \in K}$ and $\{Z_k\}_{k \in K}$: $Z_k = (E_k)^{-1}$ by (11);
7: \hspace{1.5cm} Update $\{W_k\}_{k \in K}$ by (17);
8: \hspace{1cm} until the objective function $f_4(W_k)$ converges.
9: \hspace{0.5cm} RIS Phase Shift Optimization:
10: \hspace{1cm} repeat
11: \hspace{1.5cm} Calculate $q = (\mu I - \Xi)\theta_1 - d$;
12: \hspace{1.5cm} Update $\theta_1$ by (33);
13: \hspace{1.5cm} until the objective function $f_{6b}(\theta_1)$ converges.
14: \hspace{1cm} Construct $\Theta_1$ with the obtained $\theta_1$.
15: \hspace{1cm} Optimize $\Theta_2$ in the same way as optimizing $\Theta_1$ in step 10-14.
16: until the objective function $f(W_k, \Theta_1, \Theta_2)$ converges.
Output: the optimal $\{W_k\}_{k \in K}$, $\Theta_1$ and $\Theta_2$.

D. Overall Algorithm and Complexity Analysis

Based on the above analysis, the proposed joint beamforming design is summarized in Algorithm 1, where the transmit beamforming matrix $W_k$ and RIS phase shift matrices $\Theta_1$ and $\Theta_2$ are optimized alternatively. Next, we analyze the computational complexity of Algorithm 1. For transmit beamforming design, the computational complexity is $O(I_w I_{\lambda} K N_t^2)$. For RIS phase shift design, the complexity is $O(a_{\lambda}M_1^3 + I_m M_2^3)$, where $I_w$ is the number of iterations required. Based on the above analysis, the overall computational complexity of the algorithm is $O(I_a(I_w I_{\lambda} K N_t^2 + M_1^3 + M_2^3 + I_m M_1^3 + I_m M_2^3))$, where $I_a$ is the iteration number of Algorithm 1.

IV. SIMULATION AND ANALYSIS

In this section, we evaluate the performance of the proposed algorithms. We consider a simulation scenario where the BS, RIS 1 and RIS 2 are located at (10 m, 0 m, 10 m), (0 m, 10 m, 10 m) and (10 m, 0 m, 10 m), respectively. Moreover, the users are randomly distributed in a circle centered at (10 m, 50 m, 0 m) with a radius set as 5 m. For channels, the complex gain $\rho_1$ is generated according to a complex Gaussian distribution $\rho_1 \sim CN(0, 10^{-0.15})$ with $\beta = \beta_a + 10 \beta_b \log (d) + \beta_c$. For LoS link, $\beta_a = 61.4$, $\beta_b = 5.8$ dB. For NLoS link, $\beta_a = 72$, $\beta_b = 2.92$, $\beta_c = 8.7$ dB. The generation of $q_1$ is similar to $\rho_1$. The other parameters are set as follows, where the number of transmit antennas $N_t = 8$, the number of receive antennas $N_r = 2$, the number of information streams $N_q = 2$, the total number of reflecting elements $M = M_1 + M_2 = 100$ with $M_1 = M_2 = 50$, the number of users $K = 4$, the number of paths $N_p = 5$, the noise power $\sigma^2_v = -117$ dBm, $\forall k \in K$, the transmit power $P_{\text{max}} = 30$ dBm. The following simulation results are obtained by averaging over 2000 independent channel realizations. For comparison, we consider the following schemes.

- **WMMSE+SDR**: the transmit beamforming is optimized by WMMSE method and the SDR method is used to design RISs phase shifts.
- **WMMSE+MM**: the transmit beamforming is optimized by WMMSE method and RISs phase shifts are designed via MM method.
antennas. Beamforming can be obtained with a larger number of transmit antennas. The beamforming matrix at the BS and the phase shift matrices of multi-antenna users are considered. We formulated a sum rate maximization problem by jointly optimizing the transmit beamforming matrix at the BS and the phase shift matrices of RISs. To tackle the proposed non-convex problem, we propose an effective alternating optimization algorithm by employing WMMSE, Lagrangian multiplier and MM methods. The simulation results show that the proposed algorithm achieves a good compromise between performance and computational complexity, and also verifies the great effect of double RIS on improving system performance.

V. CONCLUSIONS

In this paper, we designed a double-RIS assisted multi-user mmWave MIMO system, where the inter-RIS signal reflection and multi-antenna user are considered. We formulated a sum rate maximization problem by jointly optimizing the transmit beamforming matrix at the BS and the phase shift matrices of RISs. To tackle the proposed non-convex problem, we propose an effective alternating optimization algorithm by employing WMMSE, Lagrangian multiplier and MM methods. The simulation results show that the proposed algorithm achieves a good compromise between performance and computational complexity, and also verifies the great effect of double RIS on improving system performance.

REFERENCES


Fig. 2: The sum rate performance varies with different parameters, where (a) with the number of iterations, (b) with the transmit power $P_{\text{max}}$, (c) with the total number of reflecting elements $M$, (d) with the number of BS’s antennas $N_t$. 

- WMMSE+GD: the transmit beamforming is optimized by WMMSE method and RISs phase shifts are designed via GD method, which is proposed in [12].
- Fixed RIS: the transmit beamforming is optimized by WMMSE method while RISs phase shifts are selected randomly.

Fig. 2(a) shows the convergence behavior of all algorithms. As can be seen from the figure, the performance of WMMSE+MM is only slightly worse than that of WMMSE+SDR, but the computational complexity of WMMSE+MM is lower. Moreover, Fixed RIS has the worst performance, which indicates the importance of optimizing the RIS phase shift matrix to improve system performance. Fig. 2(b) compares the sum rate by different schemes versus the transmit power $P_{\text{max}}$ with $M = 100$, $N_t = 8$. The sum rate for all algorithms increases with $P_{\text{max}}$. It is worth noting that when the transmit power is low, the Single RIS case shows better performance, but as the transmit power increases, the performance of the Double RIS cases, i.e. WMMSE+SDR and WMMSE+MM, is significantly better than the Single RIS case and the gap between them magnifies with the increase of $P_{\text{max}}$. The impact of the total number of reflecting elements $M$ is shown in Fig. 2(c). It can be seen that the sum rate increases with $M$. The reason is that a better passive beamforming gain can be obtained with a larger number of reflecting elements, thereby improving the system performance. Furthermore, comparing Double-RIS with Single-RIS, it can be observed that Double-RIS provides a greater improvement in system performance. This is because Double-RIS can obtain significant passive beamforming gain from both double-reflection links and single-reflection links. Finally, we show the sum rate versus the number of BS’s antennas $N_t$ in Fig. 2(d). From the figure, we can see that the sum rate increases with $N_t$ since a more efficient active beamforming can be obtained with a larger number of transmit antennas.